Engaging Mathematics, Volume I: Precalculus

REGION 4

Engaging Mathematics, Volume I: Precalculus

Teacher Edition

Product ID
407-2138

Region 4 Education Service Center supports student achievement by providing educational products and services that focus on excellent, equitable outcomes for all children.

Published by
Region 4 Education Service Center
7145 West Tidwell Road
Houston, Texas 77092-2096
www.esc4.net

ISBN: 978-1-950577-59-0

Printed in the United States of America

Digital Access

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Acknowledgments

Region 4 Education Service Center would like to acknowledge the talent and expertise of those who contributed to the development of this book. Their dedication to our core values of excellence in service for children made possible the creation of this resource to assist educators in providing quality, effective instruction for all students.

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What is *Engaging Mathematics, Volume I: Precalculus*?

1 An instructional resource featuring 110 Texas Essential Knowledge and Skills (TEKS)-based, classroom-ready mathematics activities that each take approximately 15 to 20 minutes to complete.

2 A TEKS-based resource that addresses each of the Precalculus TEKS. *Engaging Mathematics, Volume I* complements teachers existing resources and provides—

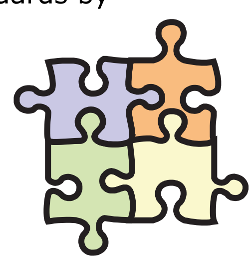
- Rigorous problem-solving tasks;
- Manipulative-based tasks;
- Vocabulary development tasks; and
- Sorting and classifying tasks.

3 A resource that supports high-quality, research-based practices by providing activities that can be used for various purposes, including—

- Engaging warm-ups and opening tasks that draw students into relevant and challenging mathematics;
- Instructional support for all students to help learners articulate, refine, and retain important mathematical concepts, processes, and skills;
- Short-cycle, formative assessments that provide immediate and ongoing feedback to guide instruction for the teacher and learning for the student; and
- Supplemental tasks to support intervention strategies.

4 A resource that incorporates the mathematical process standards by promoting—

- Reasoning, generalizing, and problem-solving in mathematical and real-world contexts;
- Modeling, using tools, and connecting representations;
- Analysis; and
- Communication.



What is found in an Engaging Mathematics TEKS-based activity?

Each activity addresses a specific student expectation that is reflected in the content objective.

Common classroom materials are used for ease of preparation. Materials are listed 1-per-student unless otherwise noted. Page titles for student handouts are bolded.

Key Features of Functions, Activity 4
P(2)(I)

Activity Objectives

I can describe the key features of a piecewise function.

I can describe how to determine the zero(s) of a piecewise function.

Materials

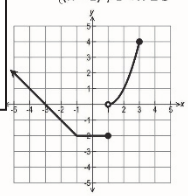
- **Piecewise Features**

It is assumed all student have access to graphing technology due to the advanced nature of the content.

ELPS have been included in the form of a student-friendly language objective.

Answer Key

$$f(x)=\begin{cases}-x-3, & x\le -1\\ -2, & -1<x\le 1\\ (x-1)^2, & 1<x\le 3\end{cases}$$



Statements

☑ The function has a domain of all real numbers less than or equal to 3.

☐ The range of the function is all real numbers.
The range of the function is all real numbers greater than or equal to −2.

☑ The function is constant on the interval $(-1,1)$.

☑ The y-intercept of the graph of the function is at −2.

☐ The function is decreasing on the interval $(-2,\infty)$.
The function is decreasing on the interval $(-\infty,-1)$.

☐ The function is increasing on the interval $(-1,4)$.
The function is increasing on the interval $(1,3)$.

☐ The function has two zeros, when $x=-3$ and when $x=1$.
The function has one zero, when $x=-3$.

☑ The graph is not symmetric about the origin.

An answer key is included for each activity.

Debriefing Question

- How did graphing the function help you determine the validity of the statements?

Communicating about Mathematics

Students may respond by recording a written response in the space provided or by talking to a partner.

Possible sentence frame:
I can determine the zeros of a piecewise function by _______.

Debriefing questions are included to assist the teacher with facilitating a post-activity

Listen For . . .

- *Use of vocabulary, such as closed interval, open interval, domain, range, symmetry, x-intercept, y-intercept, and zero.*
- *Connections between the expressions and the related domain values.*

Listen For . . .

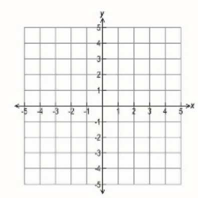
Understanding of how to determine the zeros of each part of a piecewise function from its related equation and domain values.

Date: _______________

Piecewise Features

cribe the piecewise function, *f*, by writing statements. Read curacy. Check the box if the statement is accurate. Edit ccurate.

$x\le -1$
$< x\le 1$
$1<x\le 3$



Statements

real numbe...

☐ The range of the function is all real number...

☐ The function is constant on the interval (−1...

☐ The y-intercept of the graph of the function...

☐ The function is decreasing on the interval (...

☐ The function is increasing on the interval (−...

☐ The function has two zeros, when $x=-3$ and when $x=1$.

☐ The graph is not symmetric about the origin.

Each activity includes an opportunity for students to articulate and summarize their own learning.
A sentence frame is provided for students who may need language support.

Communicating about Mathematics
How can you determine the zero(s) of a piecewise function when given the equations and their domains?

Key learning outcomes from the debriefing discussion are summarized here.

Key learning outcomes from the Communicating about Mathematics section are included here.

Graphing Functions, Activity 5

P(2)(F)

Activity Objectives

I can graph rational functions.

I can explain how to classify the horizontal asymptote of a graphed rational function.

Materials

- **Graphing Rational Functions**

Answer Key

1. g, f
2. p
3. n
4. j
5. f, p
6. p, m
7. g, m
8. j
9. n
10. m

Debriefing Questions

- How can you determine where the graph of a function is undefined?
- How can you determine if a value excluded from the domain of a rational function translates to a hole, an asymptote, or neither when graphed?
- How can you use the algebraic representation of a rational function to determine the *y*-intercept of the graph? the *x*-intercepts?

Listen For . . .

- *Use of vocabulary, such as asymptote, discontinuity, domain, hole, intercept, and range.*
- *Connection among the zeros of the denominator, zeros of the numerator, simplified expressions, the vertical asymptotes, and removable discontinuities.*
- *Connections among the degree of the numerator, the degree of the denominator, and the equation of the horizontal or oblique asymptote on the graph.*

Communicating about Mathematics

Students may respond by recording a written response in the space provided or by talking to a partner.

Possible sentence frame:
When using an algebraic representation of a rational function, ______ to determine if its graph has a horizontal asymptote. ______ to determine if its graph has an oblique asymptote.

Listen For . . .

Connections among the degree of the numerator, the degree of the denominator, and the type of asymptote on the graph.

Student Name: ______________________________ Date: ________________

Graphing Rational Functions

Determine which rational function or functions listed below would have the stated characteristic when graphed. Each stated characteristic may be found on one or more graphs.

Rational Functions		
$f(x)=\dfrac{x^2-4}{x^2-x-6}$	$g(x)=\dfrac{2}{(x-3)^2}$	$j(x)=\dfrac{(x+2)(x+1)(x-3)}{(x+3)(x-2)}$
$n(x)=\dfrac{4x^3-2x}{3x^3+x^2}$	$p(x)=\dfrac{x^2-4}{x+2}$	$m(x)=\dfrac{x^2}{x^4+1}$

__________ 1. The graph has a vertical asymptote at $x=3$.

__________ 2. The graph is a linear function with a removable discontinuity.

__________ 3. The graph has a horizontal asymptote at $x=\dfrac{4}{3}$.

__________ 4. The graph has an oblique asymptote.

__________ 5. The graph has a hole at $x=-2$.

__________ 6. The graph has no vertical asymptotes.

__________ 7. The x-axis is the horizontal asymptote of the graph.

__________ 8. The graph has a y-intercept at $(0,1)$.

__________ 9. The graph has an irrational x-intercept.

__________ 10. The domain of the graph is all real numbers.

Communicating about Mathematics
How can you use an algebraic representation to determine if a graphed rational function has a horizontal or an oblique asymptote?

____

____

____

____

____

Periodic Functions, Activity 3

P(4)(A), P(4)(C)

Activity Objectives

I can determine the sine or cosine of an angle using reference angles.

I can describe the relationship between the location of the terminal side of an angle and its sine value.

Materials

- **Reference Angles, Sine and Cosine Values**

Answer Key

1. An angle that is equivalent to $\frac{13\pi}{6}$ can be determined by subtracting 2π from $\frac{13\pi}{6}$. Since $\frac{\pi}{6}$ and the original angle are *coterminal*, the sine values are also equivalent. The sine of both $\frac{13\pi}{6}$ and $\frac{\pi}{6}$ is $\frac{1}{2}$.
2. The reference for $\frac{4\pi}{3}$ is $\frac{\pi}{3}$. The sine of the reference angle is $\frac{\sqrt{3}}{2}$. The original angle, $\frac{4\pi}{3}$, terminates in quadrant 3 . All sine values in this quadrant are negative, so $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.
3. The reference angle for $\frac{3\pi}{4}$ is $\frac{\pi}{4}$ because . . . *the terminal side of $\frac{3\pi}{4}$ forms a $\frac{\pi}{4}$ radian angle with the x-axis*.
   The cosine of $\frac{3\pi}{4}$ is $-\frac{\sqrt{2}}{2}$ because . . . $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ *and $\frac{3\pi}{4}$, terminates in the second quadrant. All cosine values in this quadrant are negative, so* $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

Debriefing Questions

- What is the relationship between an angle and its reference angle based on the concept of rotation?
- How can you determine if the sine or cosine of an angle is positive or negative?
- How can drawing an angle help you determine the quadrant of the angle and the sign of the angle's sine and cosine values?

Listen For . . .

- *Understanding that a reference angle is the measure of the acute angle that is formed with the x-axis and the terminal side of the angle in standard position. A reference angle is always positive.*

Communicating about Mathematics

Students may respond by recording a written response in the space provided or by talking to a partner.

Possible sentence frame:
The quadrant in which the terminal side of an angle lies tells us _____ .

Listen For . . .

Understanding that the sign of an angle's sine depends on the quadrant in which the terminal side lies.

Student Name: ________________________________ Date: __________________

Reference Angles, Sine and Cosine Values

Complete each paragraph to correctly describe how to determine the indicated values. Then, determine the values.

1. Determine $\sin\left(\frac{13\pi}{6}\right)$.

An angle that is equivalent to $\frac{13\pi}{6}$ can be determined by subtracting ________ from $\frac{13\pi}{6}$. Since $\frac{\pi}{6}$ and the original angle are __________, the sine values are also equivalent.

The sine of both ________ and ________ is ________.

---

2. Determine $\sin\left(\frac{4\pi}{3}\right)$.

The reference angle for $\frac{4\pi}{3}$ is ________. The sine of the reference angle is ________ . The original angle, $\frac{4\pi}{3}$, terminates in quadrant ________ .

All sine values in this quadrant are negative, so $\sin\left(\frac{4\pi}{3}\right) =$ ________.

---

3. Determine $\cos\left(\frac{3\pi}{4}\right)$.

The reference angle for $\frac{3\pi}{4}$ is _________ because . . .

The cosine of $\frac{3\pi}{4}$ is _________ because . . .

Communicating about Mathematics

What does the location of the terminal side of an angle indicate about the sine of that angle?

____

____

____

____

____